METER
ENVIRONMENT

ESTIMATION OF THERMAL STABILITY

When current flows in underground electrical cables, heat is generated. This heat must be dissipated to the environment through the soil. The cable temperature, for a given rate of heat production, is determined by the thermal conductivity of the soil, the temperature of the environment, and the geometry of the path between the cable and the environment. The thermal conductivity is strongly dependent on the water content of the soil, but heat from the cable tends to dry the soil around it, thus decreasing the thermal conductivity of the soil and increasing cable temperature. A soil in which this occurs is said to be thermally unstable. If the soil around the cable ultimately will dry fully from the heat, then the cable design needs to be done using the dry conductivity of the soil. If it is possible that it will stay wet, then higher thermal conductivities can be used in the design. Our purpose here is to do a simplified analysis to show the conditions under which thermal stability is obtained and the conditions likely to lead to thermal instability.

Analyzing linked transport of heat and water in soil can be complex (Hartley and Black, 1981; Kroener et al. 2014), but a simplified analysis at steady state conditions will be sufficient for our purposes. In the simplified analysis, we assume that water movement away from the cable is entirely in the vapor phase, due to the temperature gradient, and that water flow back toward the cable is entirely in the liquid phase, due to a matric potential gradient. We ignore the liquid flow caused by the temperature gradient and vapor flow caused by the matric potential gradient. The unsaturated hydraulic conductivity function of soil is such that there is a limiting rate of water flow for any given water content or potential. If the rate of vapor flow from the cable is greater than this limiting rate of liquid return flow, the soil will dry out. If not, it will stay wet.

LIMITING RATE OF LIQUID WATER FLOW TOWARD THE CABLE

Water flow to the cable is similar to water flow to a plant root, which was analyzed by Cowan (1965). The differential equation for this is

$$\frac{q}{A} = -k\,\frac{d\psi}{dr}$$

Equation 1

where q is flux of water to the cable (kg/s), k is hydraulic conductivity of the soil (kg s m^{-3}), ψ is the matric potential of the soil (J/kg), A is the surface area of a cylinder surrounding the cable of radius $r(2\pi rl)$ and r is the radial coordinate. The conductivity can be expressed as (Campbell, 1985)

$$k = k_e \left(\frac{\psi_e}{\psi}\right)^n$$

Equation 2

Here, the subscript e indicates the air entry point, and n is a constant ranging from 2 to 3.5. The air entry (saturated) conductivity and the air entry matric potential, as well as n, depend on soil texture and bulk density. Combining Equations 1 and 2 and integrating from the cable surface at r_c to the bulk soil at r_s gives

$$\frac{q}{2\pi\, l}\, ln \left(\frac{r_c}{r_s}\right) = \frac{k_e \psi_e^{\,n}}{1-n}\,\left(\psi_s^{\,1-n} - \psi_c^{\,1-n}\right)$$

Equation 3

As the soil gets drier, the absolute value of the matric potential gets larger (matric potential is a negative number, but for mathematical convenience, we will use absolute values here). Since n is larger than 1, the matric potential terms in Equation 3 decrease as the soil becomes drier. The limiting value for water flow occurs when the absolute value of the water potential at the cable surface is infinity and the final term in Equation 3 is zero. We can, therefore, write the limiting flux per unit length of cable (kg m⁻¹ s⁻¹) as

$$Q_{wl} = \frac{-2\pi k_e \psi_e^{\,n} \psi_s^{\,1-n}}{(n-1)\ln\left(\frac{r_s}{r_c}\right)}$$

Equation 4

VAPOR FLOW AWAY FROM THE CABLE

Fick's first law governs steady vapor transport away from the cable. We can write

$$\frac{q}{A} = -D\,\frac{dC}{dr}$$

Equation 5

where C is the vapor concentration (kg/m³) and D is the vapor diffusivity in soil. Considering just vapor movement in a temperature gradient, we can expand Equation 5 as

$$Q_{wv} = \frac{q}{l} = -2\pi r D \frac{dC}{dT} \frac{dT}{dr} = -2\pi r s D \frac{dT}{dr}$$

Equation 6

The slope of the saturation vapor density vs. temperature curve is s.

The heat flow from the cable is

$$Q_h = \frac{q_h}{l} = -2\pi r K \frac{dT}{dr}$$

Equation 7

where q_h is the rate of heat production by the cable (W), and K is the thermal conductivity of the soil. Solving Equation 7 for the temperature gradient and substituting it into Equation 6, gives

$$Q_{wv} = \frac{sDQ_h}{K}$$

Equation 8

The vapor diffusivity in soil is computed from (Campbell, 1985)

$$D = D_o b \phi^m$$

Equation 9

where ϕ is the air-filled porosity of the soil, D_o is the diffusivity in air (m²/s), and b and m are constants. Campbell (1985) gives values of 0.9 and 2.3 for b and m. The final equation for steady vapor flow from the cable is obtained by combining Equations 8 and 9.

$$Q_{wv} = \frac{s D_o b \phi^m Q_h}{K}$$

Equation 10

COMPARISON OF FLUXES TO AND FROM THE CABLE

To evaluate Equation 4, we need air entry conductivity and water potentials for representative soils. Campbell and Norman (1998) give the following values (Table 1) for typical soils. The value for n is computed as $n = 2 + 3/b$ (Campbell, 1985). If we assume the bulk soil values are measured at about three times the radius of the cable, the log term has a value around 1. The results we get are not very sensitive to this assumption, but measurements have shown that the drying effect only influences the soil close to the cable, so this seems like a reasonable value to use. Figure 1 shows results of calculations for a sand and a clay soil. Silt loam gives values about the same as clay. Other textures will be between these values.

Texture	Silt	Clay	Ψ_e J/kg	b	K_e kg s^{-1}m^{-3}
Sand	0.05	0.03	0.7	1.7	0.0058
Loamy Sand	0.12	0.07	0.9	2.1	0.0017
Loam	0.40	0.18	1.1	4.5	0.00037
Sandy Loam	0.25	0.10	1.5	3.1	0.00072
Silt Loam	0.65	0.15	2.1	4.7	0.00019
Clay Loam	0.34	0.34	2.6	5.2	0.000064
Sandy Clay Loam	0.13	0.27	2.8	4	0.00012
Sandy Clay	0.07	0.40	2.9	6	0.000033
Silty Clay Loam	0.58	0.33	3.3	6.6	0.000042
Silty Clay	0.45	0.45	3.4	7.9	0.000025
Clay	0.20	0.60	3.7	7.6	0.000017

Table 1. Hydraulic properties of soils as a function of soil texture

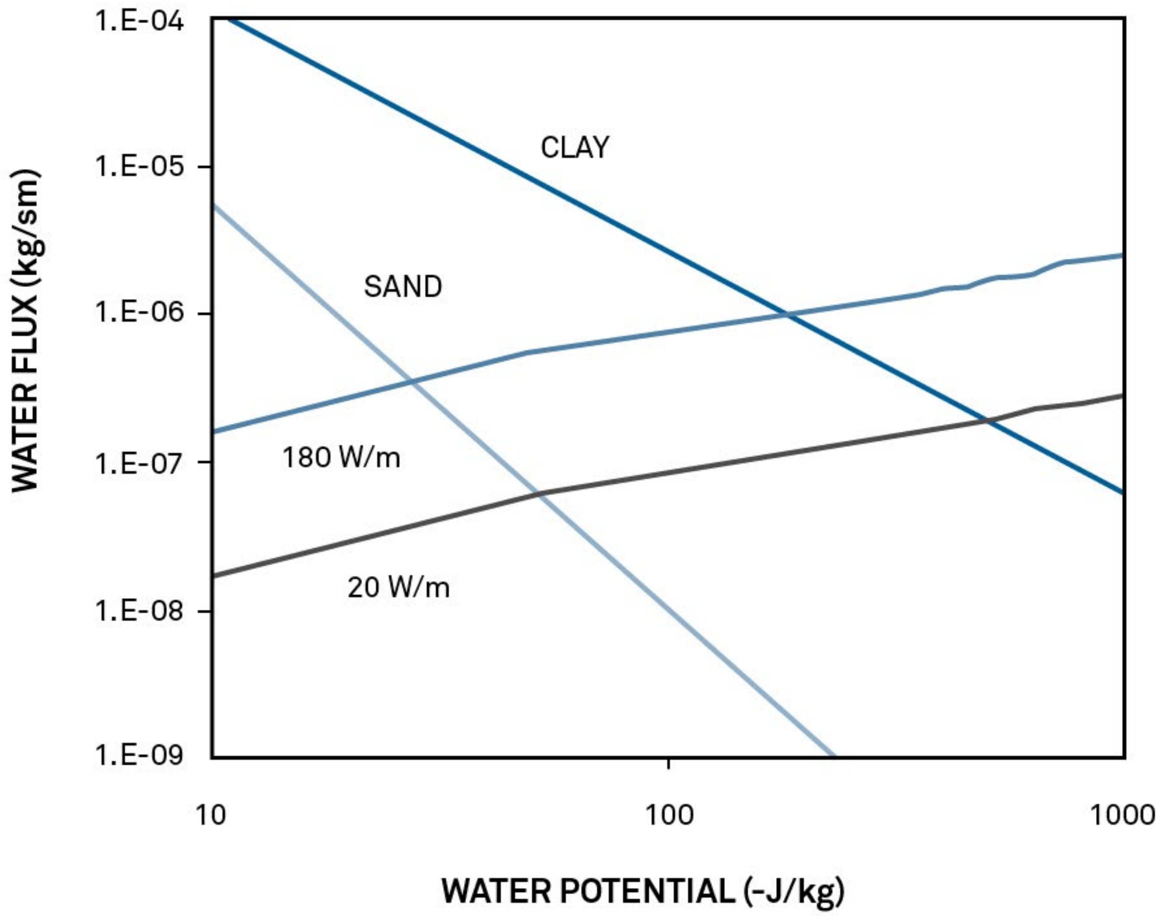

Figure 1. Graph of Equation 4 and Equation 10 for sand and clay liquid flow estimates and high and low heat dissipation rates

To plot Equation 10, we need values for ϕ, s, Q_h and K. For the order of magnitude analysis we are doing here, we could consider these quantities as being independent of water potential, though K and ϕ have a weak potential dependence. Campbell (1988) shows that the thermal conductivity of soil is independent of texture when plotted vs. water potential. From that graph, we can obtain the relationship

$$k = -6.67e - 11 \times \psi^3 + 4.35e - 7 \times \psi^2 - 1.03e - 3 \times \psi + 1.35$$

Equation 11

The air-filled porosity can be computed from (Campbell, 1985)

$$\phi = \theta_s - \theta = \theta_s \left[1 - \left(\frac{\psi_e}{\psi}\right)^{\frac{1}{b}}\right]$$

Equation 12

where θ_s is the saturation water content of the soil. The slope of the saturation vapor pressure vs. temperature function has a value of 0.001 kg/m³ at 20 °C and 0.004 at 50 °C. (Campbell, 1985). We used the larger value for our calculations, assuming the cable would be above ambient temperature. The diffusivity of vapor in air has a weak temperature dependence. We used 2.9×10-5 m2/s, which is the value at 50 °C. Limiting values for Q_h are given by Hartley and Black (1981) as 20 to 180 W/m.

Substituting these values into Equation 10 produces the vapor flux values. These differ a little between soil textures but not as much as the liquid flow value. Only the values for a mid-texture soil are shown in Figure 1 in order not to clutter the figure.

INTERPRETATION

The critical water potential is the water potential at the intersection of the vapor and limiting liquid flow lines. For sand, at 180 W/m, the critical water potential is around -10 J/kg, and at 20 W/m it's around -50 J/kg. For the clay, the critical water potentials are around -200 J/kg and -500 J/kg. To put these values in perspective, field capacity water potential is considered to be around -10 J/ kg for sands and -33 J/kg for finer-textured soils. Permanent wilt water potential is taken as -1500 J/kg. Field capacity is the water potential of the soil a few days after a heavy rain or irrigation. Water at higher potentials typically drains quickly from the profile. Permanent wilt is the water potential below which plants are no longer able to extract water. The range of plant available water is considered to be between field capacity and permanent wilt. From Figure 1, it appears that both soils would stay wet around the cable if the water potential were at field capacity, but all soils would be below critical water potential if the soil dried to permanent wilt point. Clearly, coarse-textured materials are more susceptible to thermal instability than fine-textured materials.

It would appear that soils near a water table should always be thermally stable. Soils from which plant roots have been excluded should also be thermally stable, particularly if they are periodically replenished by rain or irrigation. It appears that soils in which plants are growing would always be susceptible to thermal instability.

CAUTIONS AND KNOWLEDGE GAPS

The model we have developed here obviously needs to be checked against experiments. These likely can be carried out using heated needles, as suggested by Hartley and Black (1981). One question is the extent to which the results are influenced by enhancement of vapor flow (Cass, Campbell and Jones, 1984). Actual vapor flow could perhaps be five times larger than the value used here. Enhancement isn't well enough understood to be certain whether or not it should be included. Another effect that needs investigation is compaction. Taylor and Cavazza (1954) compare a loose and a compact soil, both at the same water content and subjected to the same temperature gradient. The loose soil is thermally unstable, while the compact soil is stable. Compacting the soil affects the thermal conductivity and porosity, but it has an even more dramatic effect on the unsaturated hydraulic conductivity. This needs to be investigated in more detail.

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