

## How large does my sample need to be for thermal properties measurements?

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We are often asked how much sample is required for thermal properties measurements. The following analysis shows how far the heat pulse travels in the soil. This gives a very conservative estimate of the required sample radius.

The rise in temperature a distance  $r$  from an infinitely long line heat source is given by:

$$\Delta T = \frac{q}{4\pi k} Ei \left( \frac{r^2}{4Dt} \right) \quad (1)$$

Where  $q$  is the heat input to the source,  $k$  is the thermal conductivity of the medium into which it is embedded,  $r$  is the distance from the source to the location of the temperature measurement,  $t$  is the time since the start of the measurement and  $D$  is the diffusivity of the medium.  $Ei$  is the exponential integral. Tabulated values exist or one can find polynomial approximations to compute its value.

The single needle probes of the KD2Pro measure the temperature of a probe of radius,  $a$ . We can use equation 1 to compute the ratio of the temperature at some distance,  $r$  from the probe to the temperature at the probe surface. The radius at which this ratio is sufficiently small, say 10%, can be taken as the radius of the sample required. The ratio is:

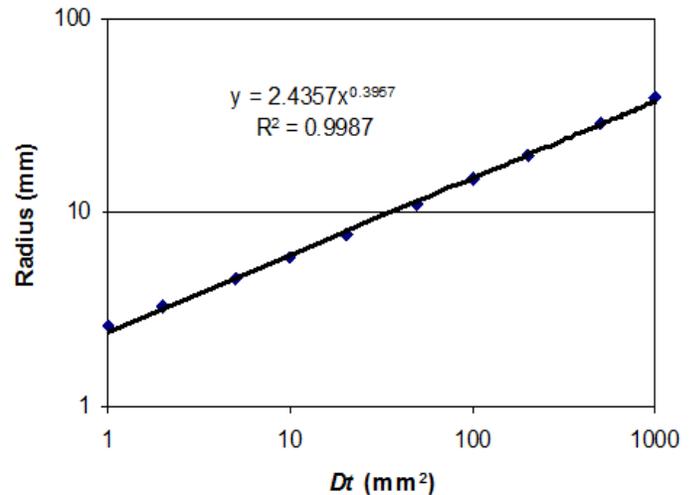
$$\frac{\Delta T}{\Delta T_a} = \frac{\frac{q}{4\pi k} Ei \left( \frac{r^2}{4Dt} \right)}{\frac{q}{4\pi k} Ei \left( \frac{a^2}{4Dt} \right)} = \frac{Ei \left( \frac{r^2}{4Dt} \right)}{Ei \left( \frac{a^2}{4Dt} \right)} \quad (2)$$

so

$$Ei \left( \frac{r^2}{4Dt} \right) = \frac{\Delta T}{\Delta T_a} Ei \left( \frac{a^2}{4Dt} \right) \quad (3)$$

This equation can't be solved explicitly for  $r$ , but a relationship between  $r$  and  $Dt$  can be obtained by iteration. The relationship is shown in the following graph. A power law equation fits the results well,

and that is also given on the graph. This graph was produced using the radius of the TR-1 needle, but the values are similar for the SH-1.



To use this you need to have some idea of the diffusivity of the material you will measure. The following table gives a few values for diffusivity along with the sample radius for the read time indicated.

Material	D (mm <sup>2</sup> s <sup>-1</sup> )	Radius (mm)			
		1 min	2 min	5 min	10 min
Water	0.14	6	8	11	14
Dry Soil	0.20	7	9	12	16
Wet Soil	0.33	8	10	15	20
Wet Sand	0.67	10	14	20	26
Delrin	0.24	7	9	13	17
HDPE	0.23	7	9	13	17
Glycerol	0.10	5	6	9	12

As an example, assume we wanted to know the required sample size for a dry powder. We estimate that the diffusivity is around 0.1 mm<sup>2</sup>/s. For a powder we want to use the longest read time possible, so we will choose 10 minutes (600 s). The product  $Dt$  is therefore 0.1 x 600 = 60 mm<sup>2</sup>. Using either the graph or the equation we would

find the radius of influence to be around 12 mm. This estimate is likely to be conservative. Smaller samples probably could be used with negligible error, but one would want to experiment with sample size to see its effect on the reading with smaller samples.