The KD2 and KD2-Pro Thermal Properties Analyzers vs. ASTM and IEEE Standards

The measurement method and analysis used by the KD2 and KD2-Pro thermal properties analyzer to obtain thermal conductivity is based on recommendations of several published standards. While the standards differ in their details, the general methods are similar for all of them. The purpose of this note is to give the background for the method, compare the KD2 and KD2-Pro hardware and measurement procedure with those of the various standards, and provide some justification for the choices made in the KD2 and KD2 Pro designs.

The method used is generally called the transient line heat source or transient heated needle method. If heat at a constant rate, q is applied to an infinitely long and infinitely small "line" source, the temperature response of the source over time can be described by the equation:

$$\Delta T = -\frac{q}{4\pi\, k} Ei\left(\frac{-r^2}{4Dt}\right) \quad (1)$$

where k is the thermal conductivity of the medium in which the line is buried, D is the thermal diffusivity of the medium, r is the distance from the line at which temperature is measured, and Ei is the exponential integral. Ei is defined in the following equation, and can be approximated by the series shown:

$$\begin{aligned} -Ei(-\alpha) &= \int_{\alpha}^{\infty} (1/u)\exp(-u)du \\ &= -\gamma - \ln\alpha + \alpha - \alpha^2/4 + ... \end{aligned} \quad (2)$$

in which $\gamma = 0.5772...$ is Euler's constant and $\alpha = r^2/4Dt$.

The terms beyond ln α in the series expansion of Ei become negligibly small for long times when r is small and D is large, so eq. 2 can be approximated as

$$\Delta T \approx \frac{q}{4\pi k}\left[-\gamma - \ln\left(\frac{r^2}{4Dt}\right)\right] = \frac{q}{4\pi k}\left[\ln t - \ln\left(\frac{r^2}{4DC_E}\right)\right] \quad (3)$$

where C_E = expγ. Thus, after some delay, a graph of ΔT vs. ln$t$ becomes a straight line with slope equal to $q/4\pi\, k$. Since two points define a straight line, k can be computed from

$$k = \frac{q(\ln t_2 - \ln t_1)}{4\pi(\Delta T_2 - \Delta T_1)} \quad (4)$$

This approximation is used in all of the transient line source standard methods for obtaining k.

Fifty years ago, when digital computers were unavailable, and computations were done by hand, there may have been some justification for such a simplified method, but there is little justification for continuing to use simplifying assumptions which produce erroneous results if the means are readily available to do better.

Approximating the exponential integral by the logarithm is one assumption made to get to eq. 4, but it isn't the only one. Real probes are neither infinitely long nor infinitely small. In addition, the ambient temperature of the sample is never constant during a measurement; there is always some temperature drift. Fortunately, the solution to the differential equation for finite length and radius probes can be obtained. For a heated cylindrical source of radius a (m) and length 2b (m), with temperature measured at its center, the temperature rise during heating is

$$\Delta T = \frac{q}{4\pi k} \int_{r^2/4Dt}^{\infty} u^{-1} \exp(-u)\exp[-(a/r)^2 u] \, I_o(2au/r) erf\left(\frac{b}{r}\sqrt{u}\right) du \tag{5}$$

Here, $I_o(x)$ represents a modified Bessel function of order zero, $erf(x)$ is the error function, and u is an integration variable. The quantity $\exp[-(a/r)^2 u] I_o(2au/r)$ approaches unity as a/r approaches 0, and $erf\left(\frac{b}{r}\sqrt{u}\right)$ approaches unity as b/r approaches infinity. In these limits, eq. 5 becomes eq. 1.

Equation 5 can be used to assess the errors which can arise by using eq. 1 or eq. 4 to obtain values for k when finite length and diameter probes are used. The construction of the KD2 and small single needle KD2-Pro probes, as well as those proposed in the standards, is consistent with an assumption that the source radius, a, and the measurement radius, r are the same. The probe lengths and diameters suggested in the various standards, as well as the dimensions Decagon's small single needle probe given in Table 1. The large single needle probe for the KD2-Pro has the same dimensions as the IEEE Lab probe. It is important to note that ASTM does not specify a needle size. The specifications given here are for a reference design in an appendix.

Table 1. Needle dimensions suggested in various standards and the Decagon needle dimensions.

	IEEE Field	IEEE Lab	ASTM	KD2
Length (mm)	2000	100	100	60
Diameter (mm)	8	2.4	1.8	1.27

Figure 1 shows the error in computing k using eq. 4 from data generated with eq. 5. The error shown is the difference between the computed k using eq. 4 and the actual k used in eq. 5 to generate the data, divided by the actual k. The time scale shows the time at

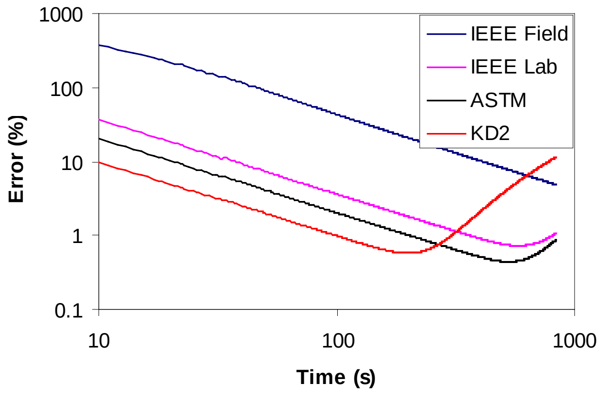

Figure 1. Error in the k value computed using eq. 4 as a function of the time at which the slope is computed for the probes shown in Table 1. Properties for the simulation were those of a dry soil. Results are similar for a saturated soil.

which the slope in eq. 4 was computed. Three things are readily apparent from the figure. First, probe size strongly affects error. The larger the probe, the larger the error at a particular time. Second, errors decrease with time, so that even large probes have acceptably small error after sufficiently long time. The third observation is that error starts to increase after sufficient heating time. This is due to the finite probe length. For an infinitely long probe, the error continues to decrease with time. All of the probes in Table 1 are sufficiently long to give negligible error from finite probe length. Finally, it is important to remember that the errors shown in Fig. 1 only occur if these probes are not calibrated (raw response used to compute k). Calibration of the probes with thermal conductivity standards eliminates this type of error.

The effect of finite probe diameter on measurement error is always in the direction of overestimating the thermal conductivity. All of the errors shown in Fig. 1 are easily eliminated by calibrating against standards of known thermal conductivity, but probes are often used without calibration. The result is that reported thermal conductivities obtained by following the procedures in the IEEE and ASTM standards are high by 30 to 50%.

Except for the field probe, acceptable error values appear to be obtained after 30 to 200 s heating. Long heating times are detrimental in at least two ways. In moist soil, water moves from regions of high temperature to regions of low temperature. The heating of the needle therefore drives moisture from around the needle. This reduces the thermal conductivity in exactly the region where conductivity is being measured. Minimizing heating time reduces the magnitude of this error.

The second effect of long heating times on error is through the effect of temperature drift on the results of the measurement. The method proposed in both the ASTM and IEEE standards is extremely susceptible to temperature drift during the measurement time. Figure 2 shows the effect on error of an extremely small sample temperature drift of –0.0001 C/s. Error is minimized by using short heat times, since the probe heats very little at long times and the effect of drift is relatively larger then.

With that background, we can now compare details of the KD2 and KD2-Pro measurement and analysis to those of the standards. The KD2 needle is shorter and smaller than suggested sizes. The large single needle of the KD2-Pro conforms to the IEEE standard, and is similar ASTM suggested dimensions. Figures 1 and 2 indicate that the smaller needle gives better results if eq. 4 is used for the analysis. In fact, if calibrations are done against standards of known conductivity, all of the needle sizes, except possibly the IEEE field probe will give accurate results.

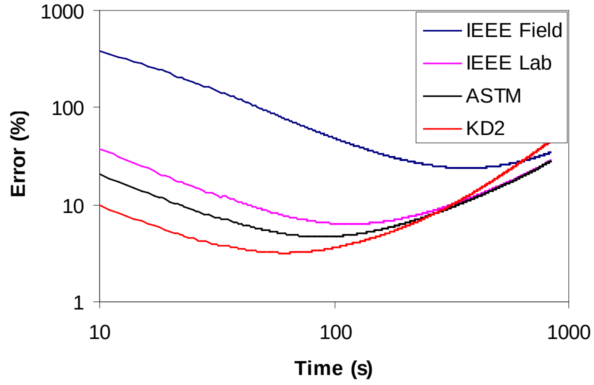

Figure 2. Error in computing *k* as a function of time of slope measurement for the probes suggested by several standards, and for the KD2 probe when there is a temperature drift in the material under test of –0.0001 C/s.

The ASTM and IEEE standards suggest collecting data with pencil and paper over a 1000 or 600 s heat time, plotting the data on semi-log graph paper, selecting a segment of the data by eye that appears to fit a straight line, selecting two points on that line to enter into eq. 4, and computing *k* from eq. 4. The KD2 and KD2-Pro collect data at 1 s intervals during a 30 s heating time and a 30 s cooling time. In the KD2 the final 20 points during heating and cooling are used in a simultaneous least squares computation which determines *k*, while removing effects of temperature drift during the measurement. Temperature is measured by a 16 bit A to D converter. In the KD2-Pro the exponential integral solution is fit to all of the data for both heating and cooling by non linear least squares, again allowing for temperature drift. Measurements of temperature are by a 24 bit A to D converter. All of the computations are done by an internal 16 bit microcontroller, and the result is displayed. Because all the computations are done internally, there is no need to record individual temperature values, forty or sixty data points are used to determine the value of *k* rather than just 2, linear temperature drift effects are removed, and subjectivity inherent in manual or “eye” fitting of data is removed. The accuracy of the measurement is verified using thermal conductivity standards such as glycerol and agar-stabilized water whose conductivity is known.

Decagon's claim that the KD2 and KD2-Pro conform to ASTM D5334 and IEEE 442 standards is based on the fact that they use a transient line heat source or transient heated needle method which uses an approximation to the solution to the differential equation for an infinite line heat source as the method for finding *k*. With the KD2-Pro the needle dimensions are similar to those specified in the standard. The fact that the analysis is done within a microcontroller, is the expected results of improved understanding of the physics, and improved technology. They only improve on the basic specified method.

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